The Small World of Seismic Events

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The understanding of long-distance relations between seismic activities has for long been of interest to seismologists and geologists. In this paper we have used data from the world-wide earthquake catalog for the period between 1972 and 2011, to generate a network of sites around the world for earthquakes with magnitude $m \geq 4.5$ in the Richter scale. After the network construction, we have analyzed the results under two viewpoints. Firstly, in contrast to previous works, which have considered just small areas, we showed that the best fitting for networks of seismic events is not a pure power law, but a power law with exponential cutoff. We also have found that the global network presents small-world properties. Secondly, we have found that the time intervals between successive earthquakes have a cumulative probability distribution explained by the non-extensive statistical mechanics. The implications of our results are discussed.

INTRODUCTION

The general belief in seismic theory is that the relationship between events that are located far apart is hard to be understood. However we live today in a world where data is being collected on most aspects of our lives and better yet, computer power is cheaply available for analyzing the data. When we put together the computer power and the data we open a series of possibilities to look for patterns in the data. The work on seismic data analysis is no different; we have now large collections of millions of seismic events from around the world which deserves analysis. In this paper we demonstrate that we live in a small world when it comes to these relations between seismic events. An event in a particular geographical site appears to be related to many other sites around the world and not only other events at nearby sites.

The ability to find useful information from data is not new and is commonly known as Data Mining. However, since the work from Barabási and Albert [1] researchers have turned their attention not on mining the data itself but rather organizing the data in a network which captures relationships between pieces of data and only then mining the network structure and hence the relations between pieces of data. The network can review information that could not possibly be seen from mining the raw pieces of data. The use of networks as a framework for the understanding of natural phenomena is nowadays called *Network Sciences* or *Complex Networks*.

In the last few years, some analysis related seismic phenomena demonstrate that earthquakes display features explained from the perspective of non-extensive statistical mechanics [2–5]. These networks display complex features that can be better understood statistically using the Tsallis entropy [6]. Through the analysis the spatial distances and the time intervals between successive earthquakes using non-extensive statistical mechanics, the authors have found that two successive earthquakes are indivisibly correlated, no matter how spatially far away they are each other [7].

In line with the successive earthquake model mentioned above, recent studies [8, 9] have applied concepts of complex networks to study the relationship between seismic events. In these studies, networks of geographical sites are constructed by choosing a region of the world (e.g. Iran, California) and its respective earthquake catalog. The region is then divided into small cubic cells, where a cell will become a node of the network if an earthquake occurred therein. Two different cells will be connected by a directed edge when two successive earthquakes occurred in these respective cells. If two earthquakes occur in the same cell we have a loop, i.e., the cell is connected to itself. Fig. 1 depicts a toy example of a network being formed. This method of describing the complexity of seismic phenomena has found that, at least for some regions, the common features of complex networks (e.g. scale-free, small-world) are present. However, in spite of the importance of the results which show that seismic networks for some specific regions present small-world effects, these results are somewhat expected, since makes sense that areas located geographically near each other, are correlated.

In this paper we have used data from the world-wide

earthquake catalog for the period between 1972 and 2011, to generate a network of sites around the world. Since only seismic events with $m \geq 4.5$ are recorded for any location in the world, we then consider these *significant events* and used them in our analysis. The results were analyzed under two viewpoints. The first, under the perspective of the complex networks theories, and the second using non-extensive statistical mechanics.

THEORETICAL BACKGROUND

Complex Networks Features

Scale-free networks are defined as one in which the degree distribution of the nodes (or vertices) in the network follows a power law, that is, the probability that a network will have nodes of degree k, denoted by $P(k)$ is given by

$$P(k) \sim k^{-\gamma}, \tag{1}$$

where γ is a positive constant.

Eq. 1 states that scale-free networks have a very small number of highly-connected nodes (called hubs) and a large number of nodes with low connectivity. These networks exist in contrast with general random networks in which where the probability distribution follows a Poisson distribution for networks with the number of nodes very large as in

$$P(k) = \binom{N}{k} p^k (1-p)^{N-k} \simeq \frac{\langle k \rangle^k e^{-\langle k \rangle}}{k!}, \tag{2}$$

where N is the number of nodes in the network and each node has an average of $\langle k \rangle$ connections. p represents the probability of an edge to be present in the network and can be shown to be approximately $\langle k \rangle / N$. Random networks have nice properties but the truth of the matter is that most real networks are not random.

The definition of a small-world network is yet to be formalized. One of the best approaches for defining small-world networks is based on the work of Watts [10] which states that in small-world networks, every node is "close" to every other node in the network. It is generally agreed that "close" refers to the average path length in the network, ℓ, which has the same order of magnitude as the logarithm of the number of nodes, i.e.,

$$\ell \sim \ln N. \tag{3}$$

In addition, and what makes small world even more interesting, is the fact that these networks have a high degree of clustering representing a transitivity in the relation of nodes; if a node i has two connections, the theory argues that the two connections are also likely to know each other. More formally, the clustering coefficient, C_i, of that node is given by:

$$C_i = \frac{\triangle(i)}{\triangle_{all}(i)} \tag{4}$$

where, $\triangle(i)$ is the number of the directed triangles formed by i with its neighbors and $\triangle_{all}(i)$ is the number of all possible triangles that i could form with its neighbors; the clustering coefficient of the entire network, C, is just the average of all C_i over the number of nodes in the network, N. In random networks the clustering coefficient can be estimated using the closed form

$$C_{\text{rand}} = \frac{\langle k \rangle}{N}, \tag{5}$$

where $\langle k \rangle$ is the average degree in the random network.

Last, one needs to understand the relation of these two characteristic to the world of seismic events. If a network of seismic events contains hubs, one can argue that the distribution of earthquakes per country around should also follow a power law. On the other hand, if the network of seismic events has small-world properties one can argue that there is some indication of long-range relations between far-apart earthquake sites.

Summary on Non-extensive Statistical Mechanics

Nonextensive statistical mechanics is a theory that explains many physical systems where the traditional Boltzmann-Gibbs statistical mechanics is insufficient. The nonextensive statistics can explain a variety of complex systems with characteristics such as long-range interaction between its elements, long-range temporal memory, fractal evolution of phase space, and certain kinds of energy dissipation. These systems have a nonextensive entropy, i.e., the entropy of the whole system can be greater or smaller than the sum of the entropies of its parts. In these cases, we use Tsallis entropy [6], which is a generalization of the Boltzmann-Gibbs entropy (defined later in this paper). The Tsallis entropy is defined as:

$$S_q = K \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1}, \tag{6}$$

where

$$\sum_{i=1}^{W} p_i^q = 1, \tag{7}$$

and W is the total number of possible configurations, p_i is the associated probabilities, K is a conventional positive constant, and q is the entropic index that can be understood as a measure of the nonextensivity of the system.

This entropy violates the additivity property, i.e. if we have two systems statistically independent A and B,

$$S_q(A+B) = S_q(A) + S_q(B) + \frac{(1-q)}{K} S_q(A) S_q(B), \quad (8)$$

where we can see that the quantity $|1-q|$ gives the measure of nonextensivity of the system under investigation.

Taking the limit $q \to 1$ in Eqs. 6 and 8, we get respectively the standard Boltzamann-Gibbs entropy, $S = -K\sum_{i=1}^{W} p_i \ln p_i$, and the additivity (extensivity) property. It is also known that applying the maximum entropy principle to the Tsallis entropy, the probability distribution obtained has a q-exponential form [11], where the q-exponential function, $e_q(x)$, is defined by,

$$e_q(x) = \begin{cases} [1+(1-q)x]^{1/(1-q)} & \text{if} \quad [1+(1-q)x] \geq 0 \\ 0 & \text{if} \quad [1+(1-q)x] < 0 \end{cases} \quad (9)$$

The inverse function of the q-exponential is the q-logarithmic function,

$$ln_q(x) = \frac{x^{1-q}-1}{1-q}, \quad (10)$$

where, for the limit $q \to 1$ in Eqs. 9 and 10 we get back the standard exponential and logarithmic functions, respectively.

A GEOGRAPHICAL NETWORK FROM SEISMIC EVENTS

The use of networks to understand phenomena associated with geographical locations has been used in many instances in science including diseases [12], scientific collaborations [13, 14], and organ transplantation [15] to mention but a few. Seismic activity intrinsically linked to geography because todays instruments can pinpoint with great accuracy the location in the globe where each seismic event take place.

It is important to precisely locate the geographical location of a seismic event but if we want to understand relations between events we should concentrate on creating a network in which locations are liked based on an acceptable criteria. In this paper we use the same procedure employed by [8] in their studies of earthquakes in specific regions of the world. The construction of the network is as follows. We first have to decide on what should represent by the nodes. Obviously our first choice are the sites where the earthquake took place. The problem of doing this is that an earthquake epicenter is rarely located exactly in the same location and given the accuracy of today's instruments we would have an infinitely large number of possible sites. We decide instead to define nodes representing a larger region of the world we here call a cell. A cell will become a node of the network if an earthquake has its epicenter therein. The creation of edges follows a temporal order of seismic events. For instance, if an earthquake occurs in a cell C_1 and the next earthquake in a cell C_2, we assume a relation between C_1 and C_2 and we represent by the event by a directed edge in the network. The process continues linking cells according to the temporal order. Fig. 1 depicts the process used to create the network from seismic events. It is worth noting that if two successive earthquakes occurs in the same cell, this node will be connected to itself via a self-edge or a loop.

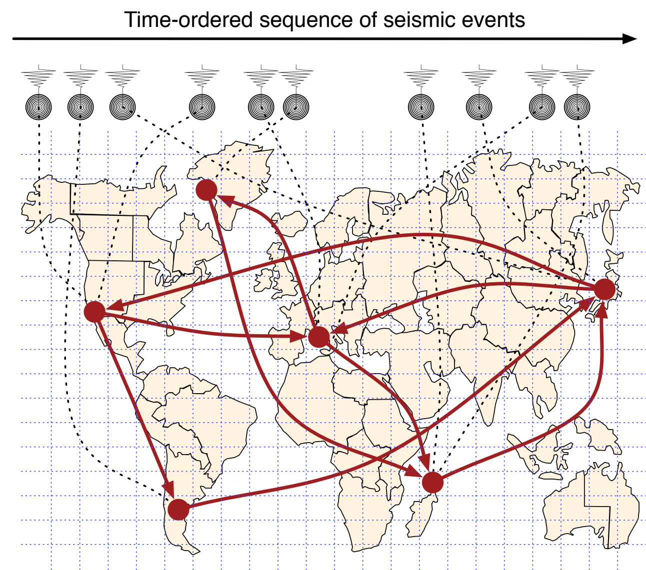


FIG. 1. This figure shows how the network of seismic events is created. At the top of the figure we see a sequence of time-ordered seismic events. Since each event has an epicenter E with location (θ_E, ϕ_E) we can use Equation 11 to calculate which cell in the map to be used as a node in the network. The nodes are linked based on the sequence of events shown at the top of the picture.

The degree of the nodes (the total number of its connections) is not affected by the direction of the network. The nature of the way the network is constructed means that for each node in the network, its in-degree is equal to its out-degree (the exceptions are only the first and last sites in the sequence of seismic events but for all practical purposes we can disregard this small difference).

Although the use of temporal ordering of events is not new in our paper, there are two main differences between our study and others. Firstly and most importantly, the region considered in our investigation is the entire globe, instead just some specific geographical subareas of the globe; this is the first worldwide study of seismic events using networks and consequently the first one to investigate the possibility of long-range links between seismic events. Secondly, we have used a two dimensional model which the depth dimension of the earthquake epicenter is not considered, since we are interested in looking for

spatial connections between different regions around the world and besides 82% of the earthquakes, in our dataset, have their hypocenters in a depth less or equal to 100 km.

Before we divide the globe into cells, we need to choose the size of such cells particularly because we are dealing with the entire globe; if the cells are too small we will not have any useful information in the network, if the cells are large we lose information due to the grouping of events into a single network node. There are no rules to define the sizes. Therefore we have taken three different sizes, the same sizes used in previous studies [8, 16], where the authors conducted studies about earthquake networks using data from California, Chile and Japan. The quadratic cells have, 5 km × 5 km, 10 km × 10 km and 20 km × 20 km. To set up cells around the globe, we have used the latitude and longitude coordinates of each epicenter in relation to the origin of the coordinates, i.e., where both latitude and longitude are equal to zero (we have chosen the referential at the origin for simplicity). So, if a seismic event occurs with epicenter E with location (θ_E, ϕ_E), where θ_E and ϕ_E are the values of latitude and longitude in radians of the epicenter, we are able to find the distances north–south and east–west between this point and the origin. These distances can be calculated, considering the spherical approximation for the Earth, by:

$$\begin{aligned} S_E^{ns} &= R.\theta_E \\ S_E^{ew} &= R.\phi_E.\cos\theta_E, \end{aligned} \tag{11}$$

where S_E^{ns} and S_E^{ew} are, the north-south and east-west distances for the earthquake E, respectively, and R is the Earth radius, considered equal to 6.371×10^3 km. With this computation we can identify the cell in the lattice for each event using the values of S_E^{ns} and S_E^{ew}. Fig. 1 shows the globe divided into a square lattice where each square in the lattice is a potential node in the network.

The seismic data used to build our network was taken from the Global Earthquake Catalog, provided by Advanced National Seismic System[17], and it records events from the entire globe. The data spans all seismic events between the period from January 1, 1972 to December 31, 2011. This catalog has a limitation because it is not consistent in all regions of the world; it includes events of all magnitudes for the United States of America but only events with $m \geq 4.5$ (in the Richter scale) for the rest of the world (unless they received specific information that the event was felt or caused damage). Therefore, in order to obtain a more homogeneous distribution of data through the world, we have analyzed only events with $m \geq 4.5$ and we also excluded data that represent artificial seismic events ("quarry blasts"). In the end, we were left with 185 747 total events, where 82% of them happen near the surface of the world (depth $\leq$ 100 km).

RESULTS

Given the network build as described in Section , we have performed a few experiments to understand this structure. Following the procedure depicted in Fig. 1, the 185 747 yielded three different networks depending on the size used for the cells. Table I presents the sizes of the three networks.

TABLE I. Three networks have been created from 185 747 seismic events in our dataset. N represents the number of nodes and M represents the number of edges. Since we have a network constructed from consecutive seismic events, the number of edges, M, is always 1 less than the number of events.

Network	N	M
20 km × 20 km	65 355	185 746
10 km × 10 km	104 516	185 746
5 km × 5 km	144 974	185 746

Scale-Free Property of the Seismic Network

It has been shown the past by the work of Abe [7, 16] that seismic networks for specific regions of the globe (e.g. California) appear to have scale-free properties, or in other words that the construction of the network employs preferential attachment as described by [1] insofar that a node added to the network has a higher probability to be connected to an existing node that already has a large number of connections. This is somewhat trivial to understand because active sites in the world will tend to appear in the temporal sequence of seismic events many times. The preferential attachment states that the probability P that a new node i will be linked to an existing node j, depends on the degree $\deg(j)$ of the node j, that is, $P(i \to j) = \deg(j)/\sum_u \deg(u)$. This rule generates a scale-free behavior whose connectivity distribution follows a power-law with a negative exponent as shown in Eq. 1.

In [7, 16], earthquake networks were built for some specific regions (California, Chile and Japan), and their connectivity distribution was defined as following a power-law. However, if we look carefully to the connectivity distribution and plot the cumulative probability of it, instead the probability density, we can observe that the power-law distributions that emerge from these network are truncated. According with [18], there are two classes of factors that may affect the preferential attachment and consequently the scale-free degree distribution: the aging of the nodes and the cost of adding links to the nodes (or the limited capacity of a node). The aging effect means that even a highly connected node may, eventually, stop

receiving new links as it normally occurs in scientific collaboration networks [19] where scientists with time stop forming new collaborations maybe due to retirement or because they are already satisfied with the number of collaborators they have. The second factor that affects the preferential attachment occurs when the number of possible links attaching to a node is limited by physical factors or when this node has, for any reason, a limited capacity to receive connections, like in a network of world airports. Thus, these two factors impose a constraint to the preferential attachment and its power-law distribution. When these factors are present, the distribution is better represented with a power law with an exponential cutoff,

$$P(k) \sim k^{-\alpha} e^{-k/k_c}, \tag{12}$$

where α and k_c are constants.

In Fig. 2, we plot the cumulative probability distribution for the earthquake network built for the Southern California (32°N − 37°N and 114°W − 122°W), using the data catalog provided by Advanced National Seismic System, where we considered all seisms with magnitude $m > 0$ for the period between January 1, 2002 and December 31, 2011. The total number of events are 147 435. It is possible to observe in this plot that the data is better fit a power-law with exponential cutoff than a pure power law which is a good fit only for small values of k with an exponent $\gamma - 1 = 0.51$, which is consistent with the value $\gamma = 1.5$ reported in [7] for the probability density function. These results apply for a network built using cell sizes of 5 km × 5 km. It is noteworthy that in a probability density plot, the cutoff does not seem to exist, because the fluctuations are higher than in a cumulative probability plot.

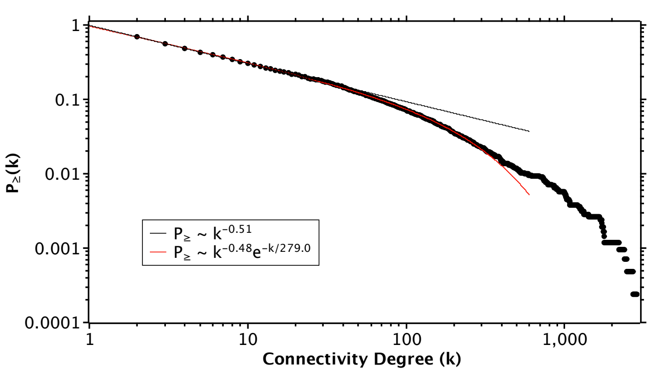


FIG. 2. Cumulative probability distribution of connectivity for the earthquake network in California using cell size 5 km × 5 km. The solid lines represent two possible fittings: a power-law (black) and a power-law with exponential cutoff (red). The best fitting is for a power-law with exponential cutoff with $\alpha = -0.48 \pm 0.0012$ and $k_c = -279.0 \pm 2.48$. There are 4 187 nodes in this network.

Before constructing the network for the entire world, we verified if the magnitude distribution of seismic events in our data has the expected behavior. The Gutenberg-Richter law gives the rate of occurrence of earthquakes with magnitude larger or equal than m,

$$F_{\geq}(m) = 10^{a-b.m}, \tag{13}$$

where, $F_{\geq}$ is the number of earthquakes with magnitude larger or equal than m and a and b are constants.

As described in [5], this equation presents problems only on small values of magnitude. Since for the globe we are using data with $m \geq 4.5$, it is expected that our magnitude data have a good agreement with the Gutenberg-Richter law. This agreement is shown in Fig. 3, which gives the cumulative distribution of magnitudes.

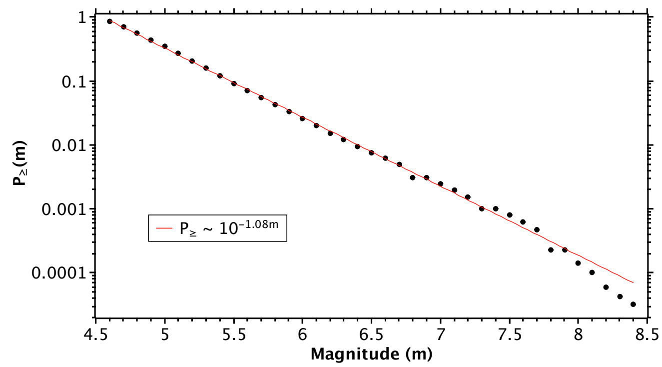


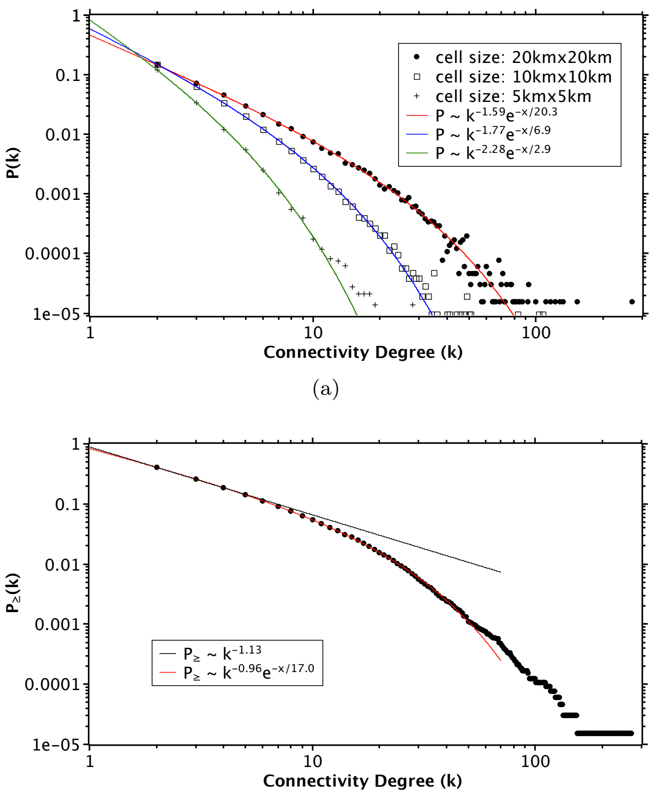
FIG. 3. Log-linear plot for cumulative probability distribution of the magnitude of earthquakes for the data used in this paper. The parameter b from the Gutenberg-Richter law (GR) has the value -1.08 ± 0.00058 and the associated value of the correlation coefficient, r, between the data (black filled circles) and the GR (red straight line) is 0.99903.

Looking at the world earthquake network constructed using the data described in Section , we note that the aging-cost effect are visibly stronger in the connectivity distribution; the exponential cutoff is clearly visible in both the degree distribution and the cumulative degree distribution in Fig. 4.

Fig. 4(a) represents the connectivity distribution for the global networks using the three different cell sizes for the global lattice. It is interesting to note that, comparing these plots, we observe that the behavior is the same in all three cases, which indicates that the cell size does not change the complex features behind the global seismic phenomena.

In Fig. 4(b), we have the same plot of Fig. 4(a), but using cumulative probability and only cell size 20 km × 20 km. Note that the cumulative probability plot for the global network shows the same exponential cutoff behavior than for local network, as shown in Fig. 2.

It is noteworthy that we did the same analysis in the connectivity distributions for the globe and for the California using different thresholds for magnitude. The



(a)

(b)

FIG. 4. Connectivity distributions in the global earthquake network. (a) Plot for the cell sizes 20 km × 20 km (solid circles), 10 km × 10 km (squares) and 5 km × 5 km (cross), where the solid lines represent the best fit using power-law with exponential cutoff. (b) Cumulative probability for the cell size 20 km × 20 km. The solid lines represent a standard power-law and (black) a power-law with exponential cutoff (red). The best fit is the power-law with exponential cutoff with $\alpha = -0.96 \pm 0.0011$ and $k_c = -17.0 \pm 0.06$.

range of thresholds for magnitude were from 4.5 to 5.5 for the globe and from 0 to 3.5 for California. As expected, in all cases the distributions have the same behavior as shown in Fig. 4.

Small-world Property of the Global Seismic Network

Small-world networks [10] have the general characteristic that they contain groups of near-cliques (dense areas of connectivity) but long jumps between these areas. These two properties lead to a network in which the average shortest path is very small and the clustering coefficient very high.

Here we would like to test if the global seismic network has small-world properties. The consequence of such finding would be an indicative that seismic events around the world are correlated and not independent. To study these properties we need to introduce slight changes to our original network. The first is that the loops have to be removed, since we are looking for correlations between nodes and it only makes sense when these nodes are different. The second change is, that we move from a network with multi-graph characteristics to a weighted network. That is, if two nodes are linked by w edges in the original network, they will be linked by a single edge with weight w in the new version of the network.

We have analyzed the seismic network for the entire world under two viewpoints: directed and undirected, where the cell size used in this construction was 20 km × 20 km. The data used were the same as described in Section . Table II shows the results obtained for the clustering coefficient (C) [20] and the average path length (ℓ) [21].

TABLE II. Results for the clustering coefficient (C) compared to the clustering coefficient of a random network of the same size (C_{rand}) and the average path length (ℓ) compared to the $\ln N$, where N is 65 355 in network with cell size 20 km × 20 km.

Network	C	C_{rand}	ℓ	$\ln N$
Directed	0.007	4.2×10^{-5}	17.19	11.08
Undirected	0.012	4.2×10^{-5}	12.24	11.08

From Table II, we note that both versions of the earthquake network has small-world properties; the clustering coefficient is much higher than an equivalent for a random network, and the average path length has the same order of magnitude as the logarithm of the number of nodes. It is worth noticing that the regional earthquake networks built for California, Japan and Chile also are small-world [7, 16] although the significance of small world at the global level is higher because we demonstrate long-range relations.

Time interval between successive seismic events

We also studied the relationship between different regions of the world under another viewpoint, which is based on the analysis of the time intervals between the successive earthquakes in our global network.

Previous studies have found that the probability distribution of time intervals between successive seismic events in small areas of the world (e.g. California and Japan) can be well described by nonextensive statistical mechanics [2, 3]. We will verify in this section if these features are still present when we look to the entire world.

In [3], the authors use concepts from nonextensive statistical mechanics to show that the cumulative probability distribution, $P_{\geq}$, for the time interval between suc-

cessive earthquakes in California and Japan follows a q-exponential,

$$P_{\geq}(\Delta t) = e_q(-\beta\Delta t), \tag{14}$$

where, Δt is the time interval between successive events and β is a positive value.

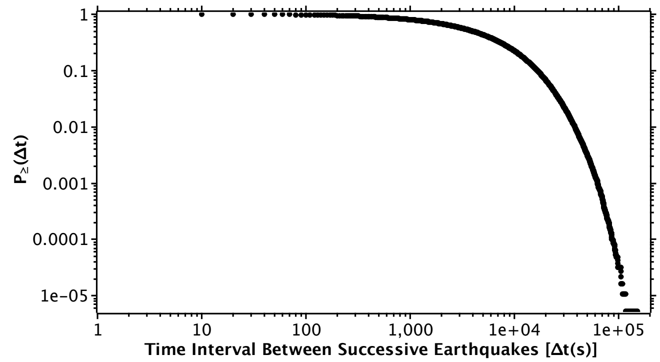
From Eqs. 9 and 14 , it is possible to see that, if $q > 1$, when $\Delta t \gg [\beta(1-q)]^{-1}$, the cumulative distribution represented in Eq. 14 approaches a power-law given by, $P_{\geq}(\Delta t) \sim \Delta t^{1/(1-q)}$.

Furthermore, from Eq. 10, if we take the ln_q in both sides in Eq. 14,

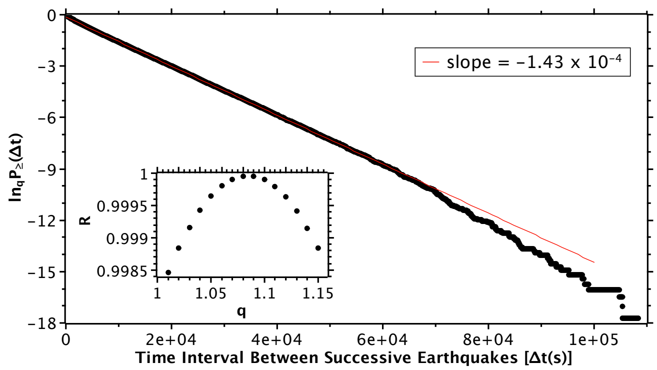
$$ln_q(P_{\geq}(\Delta t)) = -\beta\Delta t, \tag{15}$$

we can observe that the q-logarithmic of $P_{\geq}(\Delta t)$ is linear with Δt with a slope $-\beta$.

Taking the worldwide data from Section , we plotted the cumulative probability distribution for the time interval between successive earthquakes and we noticed that this distribution is well fitted by a q-exponential, indicating that the nonextensive behavior is also present when we look at seismic events from a global perspective. Fig. 5(a) shows the cumulative distribution on a log-log plot, where the histogram was made by using bins of size equal to 10 seconds. In Fig. 5(b) we have the same distribution in a q log-linear plot, where the best value of q was found by analyzing the values of the correlation coefficients, as shown in the inset of Fig. 5(b). These correlation coefficients were taken in time intervals smaller than 51 000 seconds.



(a)



(b)

FIG. 5. Cumulative distribution of the time intervals between successive earthquakes with $m \geq 4.5$ in the entire world. (a) *log-log* plot. (b) q *log-lin* plot, where the black dots represent the data and the red straight line represents the best fitting using the Eq. 15. The slope of this plot gives $\beta = 1.43 \times 10^{-4} \pm 1.89 \times 10^{-8} s^{-1}$. Inset: linear correlation coefficient for each value of q, where we can see that the best fitting is reached by $q = 1.08$.

CONCLUSIONS

The use of networks to model and study relationships between seismic events has been used in the past for small areas of the globe. Here we demonstrate that similar techniques could also be used at the global level. More importantly, many of the techniques used in complex network analysis were used here to show that there seem to exist long-distance relations between seismic events.

First we argued in favor of the long-distance relation hypothesis by showing that the network has small-world characteristics. Given the small-world characteristics of high clustering and low average path length, we were able to argue that seisms around the world appear not to be independent of each other. To strengthen this argument, we decided to do a temporal analysis of our network. Plotting the probability distribution for the time intervals between successive earthquakes, we have found that this distribution is well fitted by a q-exponential, indicating a behavior described by the non-extensive statistical mechanics, which obtain q-exponential distributions from the generalized Tsallis entropy. This non-extensive behavior also contributes to the long-distance relation hypothesis, since the non-extensive statistical has been used to explain many complex systems with long-range interactions and long-range temporal memory. Furthermore, our results contribute for the conjecture of the connections between scale-free networks and non-extensive statistical mechanics, as proposed in [22, 23].

Another interesting approach we intend to do in the future relates to using community analysis or community detection to understand how seismic locations are grouped. We believe that given the long-range relations that we found here it is unlikely that the globe would be well organized around local communities of nodes (geographical locations).

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